

# 3-D Simulations of MHD Jets - The Stability Problem

Masanori Nakamura and David L. Meier

*Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA*

**Abstract.** Non-relativistic three-dimensional magnetohydrodynamic simulations of Poynting-flux-dominated (PFD) jets are presented. Our study focuses on the propagation of strongly magnetized hypersonic but sub-Alfvénic flow ( $C_s^2 \ll V_{\text{jet}}^2 < V_A^2$ ) and the development of a current-driven (CD) kink instability. This instability may be responsible for the "wiggled" structures seen in VLBI-scale AGN jets. In the present paper we investigate the nonlinear behavior of PFD jets in a variety of external ambient magnetized gas distributions, including those with density, pressure, and temperature gradients. Our numerical results show that PFD jets can develop kink distortions in the trans-Alfvénic flow case, even when the flow itself is still strongly magnetically dominated. In the nonlinear development of the instability, a non-axisymmetric mode grows on time scales of order the Alfvén crossing time (in the jet frame) and proceeds to disrupt the kinematic and magnetic structure of the jet. Because of a large scale poloidal magnetic field in the ambient medium, the growth of surface modes (*i.e.*, MHD Kelvin-Helmholtz instabilities) is suppressed. The CD kink mode ( $m = 1$ ) grows faster than the other higher order modes ( $m > 1$ ), driven in large part by the radial component of the Lorentz force.

## INTRODUCTION

Magnetohydrodynamic (MHD) mechanisms are commonly considered to be the likely model for accelerating the wind/outflow from Young Stellar Objects (YSOs), X-ray binaries (XRBs), Active Galactic Nuclei (AGN), Microquasars, and Quasars (QSOs) [see, e.g., 1, and references therein]. The toroidal (azimuthal) component of the magnetic field, generated by the twisting of the magnetic field threading an accretion disk, plays an important role in magnetic acceleration [2]. High resolution observations of AGN show morphological structures, such as "wiggles (kinks)" or "bends", not only on kpc scales, but also pc scales and smaller [3]. Such helical distortions might be caused either by plasma instabilities or by precession of the jet ejection axis due to the gravitational interaction of binary Black Holes (BBHs) [4], BH/disk, or galaxies.

The magnetically driven jet model may be supported by recent observations. The rotation-measure (RM) distribution for the parsec-scale 3C 273 jet has a systematic gradient across the jet, and the projected magnetic field vector is systematically tilted from the jet central axis [5]. Similar RM gradients also occur in several BL Lac objects [6]. These results indicate the presence of a toroidal component of the field inside jet.

Based on both theoretical aspects and observational results, it is natural to investigate plasma instabilities that might occur in these observed morphological structures. In the MHD model the toroidal field component provides the collimation of the jet by magnetic tension or "hoop" stress. However, it is well-known that such a cylindrical plasma

column with a helical magnetic configuration is subject to MHD instabilities. These are usually divided into pressure-driven instabilities (PDIs) and current-driven instabilities (CDIs) [see, e.g., 7]. CDIs arise because the toroidal field component of a magnetically driven jet is equivalent to a poloidal (axial) electric current, and such “current-carrying” jets are susceptible to MHD instabilities. Kelvin-Helmholtz instabilities (KHIs), which are driven by velocity gradients, must be also taken into account [see, e.g., 8]. Because these above instabilities would occur together in real astrophysical situations, it might difficult to separate their individual effects.

In the recent past, much less theoretical attention has been paid by the astrophysical community to PDIs [9, 10] and CDIs [11, 12, 9] than to KHIs. KHIs have been considered for the past few decades by many theoretical or numerical investigators [for reviews, see 13, 14, and references therein]. In general, sub-Alfvénic jets ( $V_j < V_A$ ) are stable against KHIs, while, super-Alfvénic, but trans-fast-magnetosonic jets ( $V_A < V_j \sim V_{FM}$ ) are KH unstable. For super-fast-magnetosonic jets, if they have a considerably large fast-magnetosonic Mach number  $M_{FM} (\equiv V_j/V_{FM})$ , then the jets gradually become stable again. However, there are several effects that can stabilize jets beyond the Alfvén surface: (1) toroidal (rotational) velocity [15] and toroidal magnetic field [16, 17], (2) considerable mass entrainment which increases  $\eta (\equiv \rho_j/\rho_{ext.})$  [18, 19], and (3) external magnetic fields [18] or magnetized winds [20, 21]. Also, as found by [22], jets can maintain their stability by keeping the average Alfvén Mach number ( $M_A \equiv V_j/V_A$ ) within the jet to order unity by concentrating poloidal magnetic flux near the jet axis, giving it a reasonably stiff “backbone”.

Appl, Lery, & Baty [23] investigated cold super-fast-magnetosonic force-free jets ( $V_{FM} < V_j$ ). In the linear phase, the fastest growing CDI ( $m = 1$ ) is nearly independent of the radial profile of the pitch. Lery, Baty, & Appl [24] then performed numerical simulations based on their linear analysis. In the early non-linear phase, nothing in their numerical results indicate a possible disruption of jets due to CDIs ( $m = 0$  and 1). To our knowledge, only two numerical studies of pure CDIs for astrophysical jets have been performed. Todo et al. [25] applied their results to YSO jets, and Nakamura, Uchida, & Hirose [26] applied them to a large scale (kpc scale) wiggled structure of AGN jets.

In the present paper, we report 3-D non-relativistic MHD simulations of PFD jets and investigate CDIs in decreasing (density, pressure, and temperature gradients) magnetized atmospheres. PFD jets should be applicable to small scale AGN jets, such as those on parsec scales and smaller. We believe that the helical morphological features (wiggles and bends) indicate the presence of CDIs in highly magnetically dominated jets.

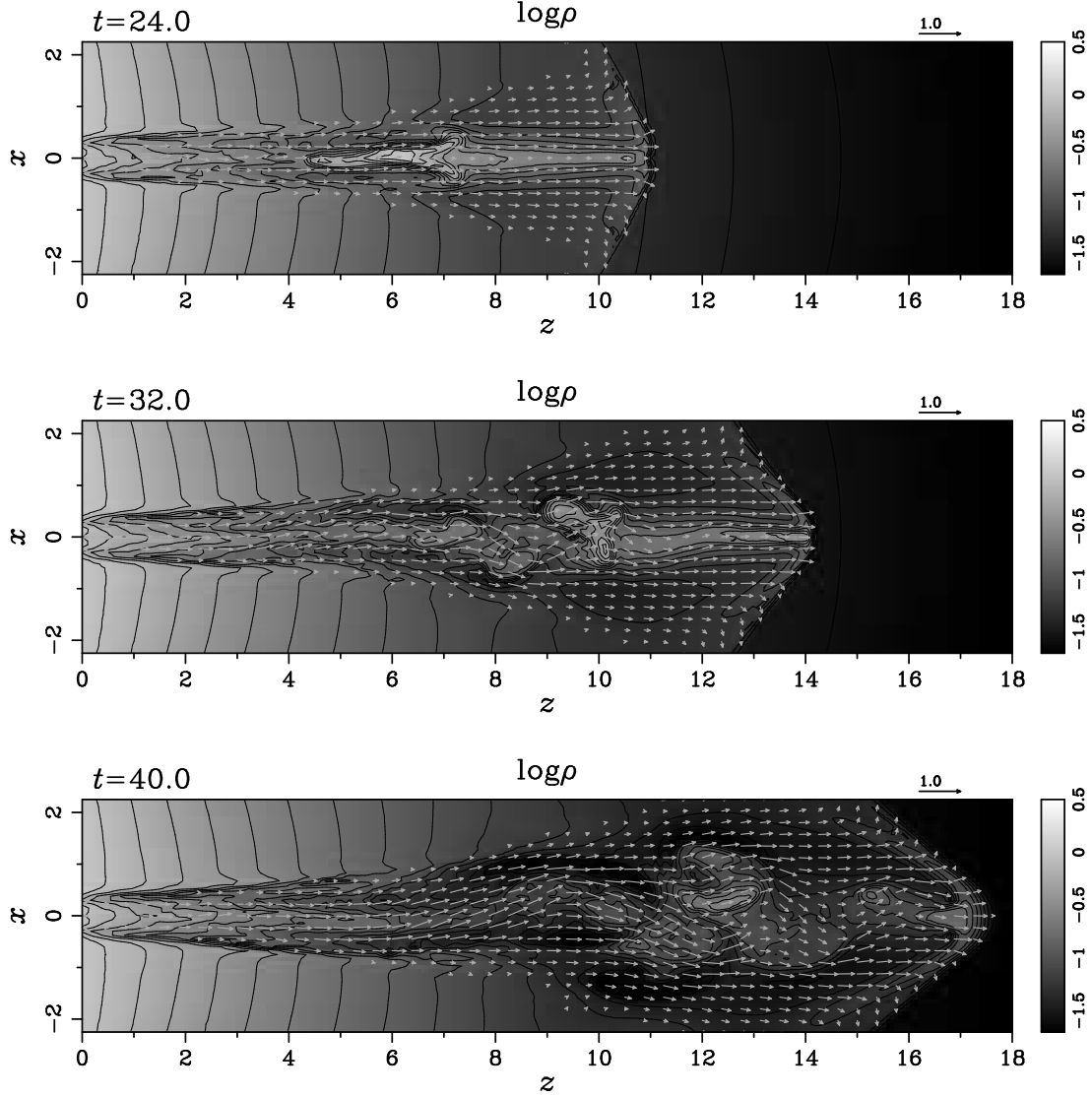
## NUMERICAL MODELS

The numerical approach in the present paper assumes non-relativistic compressible ideal MHD, with gravity neglected. We simulate the non-linear system of time-dependent MHD equations in a 3-D Cartesian coordinate system  $(x, y, z)$ . All physical quantities are normalized with a unit scale length  $L_0$ , typical density  $\rho_0$ , typical velocity  $V_0$ , and combinations thereof.  $\rho_0$  and  $V_{A0}$  are the initial value of the density and Alfvén velocity at the origin of the computational domain.  $L_0 = 2R_{j0}$  is the initial jet diameter at  $z = 0$ .

The initial distribution of the external ambient inter-galactic medium (IGM) consists of a stratified and magnetized atmosphere with possible gradients in density, pressure, and temperature. We adopt an initial current- (and therefore force-) free magnetic configuration,  $J(\equiv \nabla \times B) = 0$ , where  $B$  is the magnetic field and  $J$  the corresponding current density. Under the assumptions of flux and mass conservation, the density will be proportional to the local magnetic field as  $\rho \propto |B|^\alpha$ , where  $\alpha$  is a free parameter. For  $\alpha = 2$ , the Alfvén speed is constant throughout the computational domain, while for  $\alpha < 2$ , the Alfvén speed decreases with distance from origin and for  $\alpha > 2$  it increases. The gas pressure  $P$  is assumed to be polytropic,  $P \propto \rho^\Gamma$ , where  $\Gamma = 5/3$  is the polytropic index. The atmosphere is artificially bound to prevent it from expanding under its own pressure gradient by introducing a “pseudo-gravitational” potential designed to hold on to the atmosphere without significantly impeding the advancing jet [27]. In the lower “boundary zone” ( $z_{\min} < z < 0$ ) the boundary velocity is specified for all time as  $v_j = v_\phi(r, z)\hat{\phi} + v_z(r, z)\hat{z}$ , where  $r = (x^2 + y^2)^{1/2}$ . This represents a continuous cylindrical MHD inflow, powered by non-linear torsional Alfvén waves (TAWs) into the “evolved” region ( $z \geq 0$ ) of the computational domain. The total computational domain is taken to be  $|x| \leq x_{\max}$ ,  $|y| \leq y_{\max}$ , and  $z_{\min} \leq z \leq z_{\max}$ , where  $x_{\max}, y_{\max} \simeq 16$  ( $32R_{j0}$ ),  $z_{\min} \simeq -1$  ( $2R_{j0}$ ), and  $z_{\max} \simeq 20$  ( $40R_{j0}$ ). The numbers of grid points in the simulations reported here are  $N_x \times N_y \times N_z = 261 \times 261 \times 729$ , where the grid points are distributed non-uniformly in the  $x$ ,  $y$ , and  $z$  directions. High-resolution 3-D computations were performed on a FUJITSU VPP 5000/32R (9.6 GFLOP/s peak speed per processor) and required about 6 hours on the 32-processor machine.

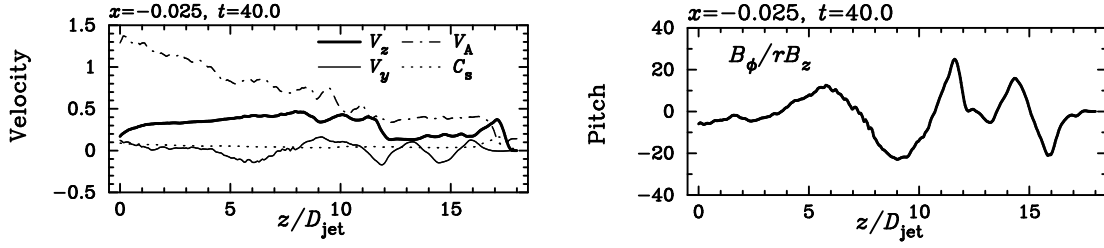
## NUMERICAL RESULTS

We concentrate on the solution in the region,  $-2.25 \leq x, y \leq 2.25$ , and  $0.0 \leq z \leq 18.0$  and discuss the dynamical behavior of a typical case: the growth of CDIs in a PFD jet, with parameter  $\alpha = 1$ .  $B_z$  and  $\rho$  decrease gradually along  $z$ -axis and tend to  $\sim z^{-2}$  for  $z > 1$ . We set the plasma beta ( $\beta_0 \equiv 2C_{s0}^2/\gamma V_{A0}^2 = 10^{-2}$ ) at the origin, and the ratio  $V_A/C_s$  increases gradually along the  $z$ -axis. Throughout the time evolution, a quasi-stationary PFD flow ( $F_{E \times B}/F_{\text{tot.}} \sim 0.9$ , where  $F_{E \times B}$  Poynting flux and  $F_{\text{tot.}}$  total energy flux) is injected into the “evolved” region from the lower “boundary” zone. Fig. 1 shows the time evolution of the density and the velocity field in the  $x - z$  plane for the typical PFD jet. In the early stage ( $t = 24.0$ ), a strongly magnetized helical jet powered by TAWs advances into the decreasing poloidally magnetized IGM, *remaining in radial force-free equilibrium* ( $J \times B \simeq 0$ ). This is precisely the transverse structure predicted analytically by [16]. The front of the TAW (a fast-mode MHD wave) is decelerated due to gradually decreasing  $V_A$ ; the accumulation of  $B_\phi$  occurs behind this wave front. The toroidal magnetic pressure gradient force  $d(B_\phi^2/8\pi)/dz$  becomes large and works effectively at this front, strongly compressing the external medium along this propagating TAW front. At later stages the front becomes super-fast magnetosonic and a bow shock (MHD fast-mode shock) is formed. The accumulation of  $B_\phi$ , due to the gradually decreasing  $V_A$ , causes a concentration of axial current  $J_z$  near central ( $z$ ) axis, and the distribution of magnetic pitch ( $|B_\phi/rB_z|$ ) become large. The Lorentz force breaks the quasi-equilibrium



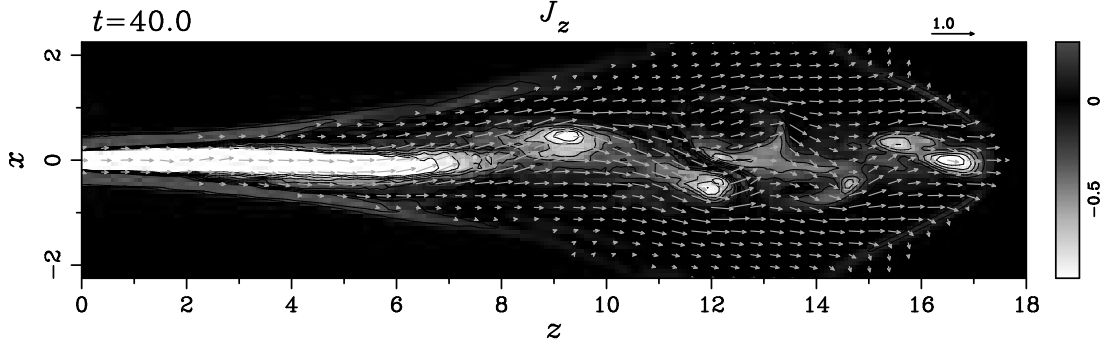
**FIGURE 1.** Time evolution of a typical PFD jet. Gray-scale images of the density distribution (logarithmic scale) are shown with the poloidal velocity field in the  $x-z$  plane at  $t = 24.0$  (*top*),  $t = 32.0$  (*middle*), and  $t = 40.0$  (*bottom*).

balance in the radial direction, and the axisymmetric PFD jet is disrupted by a CDI kink mode ( $m = 1$ ) ( $t = 32.0$ ). At late times ( $t = 40.0$ , see Fig. 1 *bottom* and Fig. 2 *right*) a large-scale wiggled structure appears, while the MHD jet is still sub-Alfvénic  $V_{\text{jet}} < V_A$  (except at the front itself; see Fig. 2 *left*). This PFD jet will continue to be subject to the CDI, even though it is magnetically dominated throughout the computational time. Fig. 3 shows the closed circulating current system, consisting of a current that is co-moving with the PFD jet (close to central axis) and a return current that flows outside the magnetized wind (*cocoon*). The external magnetized wind reduces the velocity shear between the jet itself and external medium, thereby suppressing KHIs even if the MHD



**FIGURE 2.** Distribution of velocities (*left*) and magnetic pitch (*right*) parallel to the  $z$ -axis

jets become super-Alfvénic (*i.e.*, a kinetic energy flux-dominated [KFD]).



**FIGURE 3.** Gray-scale image of the axial current density distribution in  $x - z$  plane.

## DISCUSSION

There now is observational evidence that, in at least some extragalactic systems, the AGN jets are slowly collimated over a length scale of pc ( $\gg$  BH + disk system) [e.g., M87, 28]. If the MHD process is indeed the acceleration mechanism, the Alfvén surface will be at a distance of order the disk’s outer radius  $R_{\text{disk}}$ . The observations can set an upper limit on this of a few tens of pc (the radius of the ionized disk [29]). From theoretical considerations, the optically-thick disk is expected to be at about  $R_{\text{disk}} \sim 20$  pc [30]. So, an understanding of sub-Alfvénic to trans-Alfvénic jet flow may be needed in order to understand jet dynamics and morphology on sub-pc to pc scales.

## CONCLUSIONS

We have investigated the non-linear behavior of PFD jets powered by TAWs in the extended stratified atmospheres, and find the disruption of PFD jets due to a CDI kink mode, leading to the formation of wiggles. Even if jets become super-Alfvénic during their propagation, CDI disruptions can still occur. Growing modes of MHD KHIs for super-Alfvénic flows are not seen in our parametric survey, because of external force-free magnetized helical winds surrounding the well-collimated jet, suppressing the

growth of KHIs. CDIs might be a possible origin of the wiggled morphology of AGN jets are slowly accelerated and collimated on pc scales.

## ACKNOWLEDGMENTS

We thank P. E. Hardee for useful comments and suggestions. M.N. is supported by a National Research Council Resident Research Associateship, sponsored by the National Aeronautics and Space Administration. This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract to NASA. Numerical computations were performed on the Fujitsu VPP5000/32R at the Astronomical Data Analysis Center of the National Astronomical Observatory, Japan. Data analyses and visualizations were carried out at the Supercomputing and Visualization Facility at JPL.

## REFERENCES

1. Meier, D.L., Koide, S., & Uchida, Y. 2001, *Science*, 291, 84.
2. Uchida, Y., & Shibata, K. 1985, *PASJ*, 37, 515.
3. Hummel, C.A., Schalinski, C.J., Krichbaum, T.P., Rioja, M.J., Quirrenbach, A., Witzel, A., Muxlow, T.W.B., Johnston, K.J., Matveyenko, L.I., & Shevchenko, A. 1992, *A&A*, 257, 489.
4. Begelman, M.C., Blandford, R.D., & Rees, M.J. 1980, *Nature*, 287, 307.
5. Asada, K., Inoue, M., Uchida, Y., Kamenno, S., Fujisawa, K., Iguchi, S., & Mutoh, M. 2002, *PASJ*, 54, L39.
6. Gabuzda, D.C., & Murray, E. 2003, *astro-ph/0309668*.
7. Bateman, G. 1980, *MHD Instabilities* (Cambridge: MIT Press).
8. Chandrasekhar, S. 1961, *Hydrodynamic and Hydromagnetic Stability* (New York: Oxford Univ. Press).
9. Begelman, M.C. 1998, *ApJ*, 493, 291.
10. Kersalé, E., Longaretti, P.-Y., & Pelletier, G. 2000, *A&A*, 363, 1166.
11. Eichler, D. 1993, *ApJ*, 419, 111.
12. Spruit, H.C., Foglizzo, T., & Stehle, R. 1997, *MNRAS*, 288, 333.
13. Birkinshaw, M. 1991, in Hughes P.A., ed., *Beams and Jets in Astrophysics* (Cambridge: Cambridge Univ. Press), p. 278.
14. Ferrari, A. 1998, *ARA&A*, 36, 539.
15. Bodo, G., Rosner, R., Ferrari, A., & Knobloch, E. 1996, *ApJ*, 470, 797.
16. Lind, K.R., Payne, D.G., Meier, D.L., & Blandford, R.D. 1989, *ApJ*, 344, 89.
17. Appl, S., & Camenzind, M. 1992, *A&A*, 256, 354.
18. Todo, Y., Uchida, Y., Sato, T., & Rosner, R. 1992, *PASJ*, 44, 245.
19. Rosen, A., & Hardee, P.E. 2000, *ApJ*, 542, 750.
20. Hardee, P.E., & Rosen, A. 2002, *ApJ*, 576, 204.
21. Hardee, P.E., & Hughes, P.A. 2003, *ApJ*, 583, 116.
22. Ouyed, R., Clarke, D.A., & Pudritz, R.E. 2003, *ApJ*, 582, 292.
23. Appl, S., Lery, T., & Baty, H. 2000, *A&A*, 355, 818.
24. Lery, T., Baty, H., & Appl, S. 2000, *A&A*, 355, 1201.
25. Todo, Y., Uchida, Y., Sato, T., & Rosner, R. 1993, *ApJ*, 403, 164.
26. Nakamura, M., Uchida, Y., & Hirose, S. 2001, *NewA*, 6, 61.
27. Clarke, D.A., Harris, D.E., & Carilli, C.L. 1997, *MNRAS*, 284, 981.
28. Biretta, J.A., Junor, W., & Livio, M. 2002, *NewAR*, 46, 239.
29. Ford, H.C., Harms, R.J., Tsvetanov, Z.I., Hartig, G.F., Dressel, L.L., Kriss, G.A., Bohlin, R.C., Davidsen, A.F., Margon, B., & Kochhar, A.K. 1994, *ApJ*, 435, L35.
30. Collin-Souffrin, S., & Durmont, A.M. 1990, *A&A*, 229, 292.